

Fast simulation of the Cherenkov light from showers

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Abstract

A method for fast simulation of the Cherenkov light generated by electromagnetic showers is described. The parametrization for the longitudinal profile is used and fluctuations and correlations of the parameters are taken into account in a consistent way. Our method dramatically reduces the CPU time and its results are in rather good agreement with a full Monte Carlo simulation.

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The flux of the Cherenkov photons from a shower is typically simulated by tracking all secondary particles of the shower down to the Cherenkov threshold energy E_c . For the electrons in water $E_c=0.78$ MeV. The computer time needed for simulations of this type increases linearly with shower energy. Therefore a large amount of CPU time is required for simulation of a high energy shower. However large samples of events are necessary to obtain results with a sufficient statistics. The parametrization of the longitudinal Cherenkov light profile of the showers is one of the methods to speed up simulation.

A simple algorithm for parameterized showers has been successfully used for the simulation of the UA1 calorimeter [1]. The simulation of the longitudinal energy profile of electromagnetic showers was based on fitting the parameters of a Gamma distribution to the average shower profile. Later the shape fluctuations of individual showers were systematically taken into account for the simulation of calorimeter built for the H1 experiment at HERA [2].

For a fast generation of the Cherenkov light from showers with energies $E_0 \geq 10$ GeV we have developed the program FLG. The program SIMEX [3] has been used for the full Monte Carlo (MC) simulation of the electromagnetic showers and computation of the Cherenkov light emitted by the shower particles.

2 Parametrization of electromagnetic shower

It is well known that the average longitudinal profile of electromagnetic showers is reasonably well described by the Gamma distribution

$$f(t) = \frac{dE}{dt} = E_0 \beta \frac{(\beta t)^{\alpha-1} \exp(-\beta t)}{\Gamma(\alpha)}, \quad (1)$$

where t is a shower depth in units of radiation length,

$$\int_0^{\infty} f(t) dt = E_0, \quad (2)$$

and

$$\frac{df(t)}{dt} = \left(\frac{\alpha-1}{t} - \beta \right) f(t) \quad (3)$$

The parameters β and $(\alpha - 1)/t$ are coefficients of absorption and creation of the shower particles, respectively. There is a relation between the location of the shower maximum t_{\max} and the parameters α and β ,

$$t_{\max} = (\alpha - 1)/\beta, \quad (4)$$

which follows from Eq.(1). The energy dependence of t_{\max} can be approximated by the expression

$$t_{\max} = \ln\left(\frac{E_0}{\varepsilon}\right) + C, \quad (5)$$

where $C=-0.5$ for e -induced shower, $C=0.5$ for γ - induced shower and $\varepsilon=75.5$ MeV is the critical energy for water.

The longitudinal shower profile $f(t)$ reveals a useful scaling property when the depth of a shower is expressed in units t_{\max} as follows: $\tau=t/t_{\max}$. Then Eq.(1) can be rewritten as

$$f(\tau) = \frac{dE}{d\tau} = E_0 \beta' \frac{(\beta' \tau)^{\alpha-1} \exp(-\beta' \tau)}{\Gamma(\alpha)} \quad (6)$$

and $\beta' = t_{\max} \beta$.

The total distance traveled by all charged shower particles L (track length in the units of radiation length) is proportional to the shower energy E_0 .

$$L \propto E_0/\varepsilon. \quad (7)$$

3 Parametrization of the Cherenkov light profile of shower

As we deal with underwater/underice detector measuring the Cherenkov radiation we will consider the average longitudinal Cherenkov light distribution emitted by the shower particle only. Total number of the Cherenkov photons from the shower, N_{tot}^γ , is proportional to the track length L . But according to Eq.(7) the track length is proportional to the shower energy, and we can write

$$N_{\text{tot}}^\gamma = D_\gamma E_0. \quad (8)$$

So, the average longitudinal Cherenkov light distribution $f_{\text{ch}}(\tau)$ can be parametrized as follows

$$f_{\text{ch}}(\tau) = D_\gamma f(\tau), \quad (9)$$

where

$$\int_0^{\infty} f_{\text{ch}}(\tau) d\tau = N_{\text{tot}}^{\gamma} = D_{\gamma} E_0. \quad (10)$$

Then the number of the Cherenkov photons emitted by the shower particles in the layer $\Delta\tau$ at the depth τ is

$$N(\tau, \Delta\tau) = \frac{N_{\text{tot}}^{\gamma}}{\Gamma(\alpha)} [\gamma(\alpha, \beta'(\tau + \Delta\tau)) - \gamma(\alpha, \beta'\tau)], \quad (11)$$

where

$$\gamma(\alpha, x) = \int_0^x t^{\alpha-1} \exp(-t) dt \quad (12)$$

is the incomplete Gamma function.

A full MC simulation of the electromagnetic shower in water (the radiation lenght $X_0=36.1$ cm) has been done using the program SIMEX. The value for the tracking energy threshold was set to 1 MeV. We obtain that the number of photons in the wavelength $300 \text{ nm} \leq \lambda \leq 600 \text{ nm}$ per MeV of the shower energy is

$$D_{\gamma} = N_{\text{tot}}^{\gamma}/E_0 \approx 163 \text{ photons/MeV}. \quad (13)$$

4 Simulation of the Cherenkov light profile taking into account the fluctuations of showers

Realistic simulation, however, requires the simulation of individual showers. Fluctuations of the parameters in Eq.(9) obtained from the averaged Cherenkov light profile does not necessarily lead to a correct description of the fluctuations of individual showers. Assuming that also individual shower profiles can be approximated by the Gamma distribution

$$f(\tau) = E_0 \beta'_i \frac{(\beta'_i \tau)^{\alpha_i-1} \exp(-\beta'_i \tau)}{\Gamma(\alpha_i)}, \quad (14)$$

the fluctuations and correlations can be taken into account consistently. The index i indicates that the function describes an individual shower i with the

parameters α_i and β_i . The α_i and β_i can be calculated from the first τ_i and second σ_i^2 moments of the distribution Eq.(14) for each single SIMEX-simulated shower:

$$\alpha_i = (\bar{\tau}_i/\sigma_i)^2 \quad \text{and} \quad \beta'_i = \bar{\tau}_i/\sigma_i^2. \quad (15)$$

The statistics of the full MC simulation was 1000 events for each fixed energy E_0 in the region $10 \text{ GeV} \leq E_0 \leq 10^4 \text{ GeV}$ and 200 events for $E_0=10^5 \text{ GeV}$. The parameters α_i and β'_i are normal-distributed and such that the means $\langle\alpha_i\rangle$, $\langle\beta'_i\rangle$ and their fluctuations σ_α and σ_β can be determined and parametrized as a function of the shower energy. The average values $\langle\alpha_i\rangle$ and $\langle\beta'_i\rangle$ vs shower energy are shown in Fig.1. They can be approximated by a logarithmic energy dependence. The correlation of the α_i and β'_i is given by

$$\rho = \frac{\langle (\alpha_i - \langle\alpha\rangle)(\beta'_i - \langle\beta'\rangle) \rangle}{[(\langle\alpha_i^2\rangle - \langle\alpha\rangle^2)(\langle\beta_i'^2\rangle - \langle\beta'\rangle^2)]^{1/2}} \quad (16)$$

and slowly increases with shower energy from $\rho \sim 0.77$ at $E_0=10 \text{ GeV}$ to $\rho \sim 0.85$ at $E_0=10^5 \text{ GeV}$.

In the simulation a correlated pair (α_i, β'_i) is generated to

$$\begin{pmatrix} \alpha_i \\ \beta'_i \end{pmatrix} = \begin{pmatrix} \langle\alpha\rangle \\ \langle\beta'\rangle \end{pmatrix} + B \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \quad (17)$$

with

$$B = \begin{pmatrix} \sigma_\alpha & 0 \\ 0 & \sigma_\beta \end{pmatrix} + \begin{pmatrix} \sqrt{(1+\rho)/2} & \sqrt{(1-\rho)/2} \\ \sqrt{(1+\rho)/2} & -\sqrt{(1-\rho)/2} \end{pmatrix}, \quad (18)$$

where z_1 and z_2 are standard normal-distributed random numbers.

The results of the full and the fast simulations are shown in Fig.2 and Fig.3, respectively. In these figures the normalized means longitudinal Cherenkov profiles $N(\tau, \Delta\tau)/N_{\text{tot}}^\gamma$ and their fluctuations $\sigma(\tau)/N(\tau, \Delta\tau)$ for e -induced showers with different energies are given. A comparison of the SIMEX and FLG simulations shows a good agreement in the average profiles. The agreement in the fluctuations in the depth region $0.5 \leq \tau \leq 2$ improves with an increase of shower energy.

The average angular distribution of the Cherenkov photons emitted in the layer $\Delta\tau$ at shower depth τ with respect to the shower axis can be written as follows:

$$P(\tau, \Delta\tau, \cos\theta) = N(\tau, \Delta\tau)\psi(\tau, \Delta\tau, \cos\theta) \text{ photons/ster}, \quad (19)$$

$$\text{where} \quad 2\pi \int_{-1}^1 \psi(\tau, \Delta\tau, \cos\theta) d\cos\theta = 1 \quad (20)$$

The function $\psi(\tau, \Delta\tau, \cos\theta)$ was calculated for each layer $\Delta\tau = \tau_{i+1} - \tau_i = 0.1$ by the program SIMEX and use for the fast Cherenkov light generation. The angular distribution of emitted photons averaged over the cascade depth,

$$F(\cos\theta) = \frac{1}{N_{\text{tot}}^\gamma} \int_0^\infty P(\tau, \Delta\tau, \cos\theta) d\tau, \quad (21)$$

depends very slowly on the shower energy and is shown in Fig.4 vs $\cos\theta$ for the different values of E_0 . In this figure the result obtained in Ref.[4] is also given for comparison. The angular dependence of $\psi(\tau, \Delta\tau, \cos\theta)$ for different shower depths is shown in Fig.5. Note that the width of the $\psi(\tau, \Delta\tau, \cos\theta)$ distribution increases and the average value of $\cos\theta$ decreases with τ .

5 Fast generation of the Cherenkov light from shower and hight energy muons

A procedure of the FLGf generation of the Cherenkov photons emitted by a shower is the following. The shower is divided into layers with the size $\Delta\tau = 0.1$. Each layer is considered as the point-like source of light. Then the flux of the Cherenkov photons from the i -th layer at some point X is

$$\Phi_i = P(\tau_i, \Delta\tau, \cos\theta) / R_i^2, \quad (22)$$

where R_i is the distance between the center of the i -th layer and the point X , and θ is the angle between the shower axis and the direction from the center of the i -th layer to the point X . The total light flux at the point X emitted by the entire shower is obtained as the sum of contributions from each shower layer.

To check correctness of the simulation using the program FLG, the number of photoelectrons collected in the PMT (Hamamatsu R2018) has been compared with that from the full MC simulation. The detector configuration simulated for the comparison is given in Fig.(6). The number of the photoelectrons N_{pe} multiplied by the distance squared R^2 between the location of the shower maximum and the PMT is shown in Fig.(6) for different orientations of the PMT with respect to a shower with energy 1 TeV. The comparison of the FLG and the full MC simulation reveals that:

- the results are in a good agreement
- N_{pe} depends very much on the PMT direction with respect to the shower
- at the distance $R > 20$ m the shower with energy 1 TeV can be considered as a point-like source of light.

The CPU time for the full simulation of the Cherenkov light is large and increases with the shower energy. But when we use the program FLG the CPU time does not depend on the shower energy and decreases dramatically. For example, at the energy $E_0=10$ GeV the CPU time for the FLG simulation is $3 \cdot 10^2$ times smaller than for the full MC simulation. At the shower energy $E_0=10^5$ GeV the gain is $\approx 3 \cdot 10^6$ times.

The muon undergoes continuous (ionization) and stochastic energy loss processes (δ -electrons, bremsstrahlung, e -pair production and muon nuclear interaction). In our program for muon tracking the secondary processes below 0.01 GeV are not simulated individually, but treated as quasi-continuous energy loss. The additional light flux N_a from these low energy processes is parametrized by

$$N_a = N_0(a + b \ln E_\mu). \quad (23)$$

Here N_0 is the Cherenkov light from the 'naked' muon itself with energy E_μ . The full MC simulation is used for calculation of the Cherenkov light from the secondary particles with energies $0.01 < E < 10$ GeV. The flux of the Cherenkov photons emitted by secondaries with energies $E \geq 10$ GeV is simulated by the program FLG.

6 Conclusions

A full simulation of secondary electromagnetic processes requires a large amount of CPU time, in particular when a low energy threshold for the tracking (≈ 1 MeV) has to be used and when large detector volumes are considered. In order to speed up the simulation the program FLG has been developed. It provides a realistic and fast simulation of the Cherenkov light from electromagnetic showers. Shower to shower fluctuations and correlations are taken into account consistently. The agreement between the FLG results and the full MC is quite good. The CPU time has been dramatically reduced with the FLG simulation. At the shower energy 10^5 GeV the gain in CPU time is $\approx 3 \cdot 10^6$ times.

The program for the muon tracking and the FLG codes provide a fast and

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References

- [1] R.K. Bock et al. Nucl. Instrum. Meth. 185 (1981) 533
- [2] G. Grindhammer, M. Rudowicz, and S. Peters, Nucl. Instrum. Meth. A290 (1990) 469
- [3] B. Stern Preprint INR P-0081,P-0082 (1978)
- [4] A. Belayev, I. Ivanenko, and V. Makarov Proc. of the 1978 DUMAND Summer Workshop. Editor: A. Roberts 1 (1979) 337

Fig.1: The parameters $\langle\alpha\rangle$ and $\langle\beta'\rangle$ of the Gamma distribution vs shower energy. The point bars are σ_α and σ_β .

Fig.2: The normalized mean longitudinal profiles of the Cherenkov light $N(\tau, \Delta\tau)/N_{\text{tot}}$ for e -induced showers with the energies $E_0 = 10, 10^3$ and 10^5 GeV.

Fig.3: The relative variance $\sigma(\tau)/N(\tau, \Delta\tau)$ of the Cherenkov light profiles for e -induced shower with energies $E_0 = 10, 10^3$ and 10^5 GeV.

Fig.4: The normalized integral angular distribution of the Cherenkov photons for different shower energies.

Fig.5: The normalized differential angular distribution of the Cherenkov photons at different shower depths.

Fig.6: $R^2 * N_{\text{p.e.}}$ as a function of the distance R between the location of the shower maximum and the PMT for different directions of the PMT with respect to the maximum (bold star) at the shower energy $E=1000$ GeV. Arrows show the orientations of the PMT: $V=\cos(\theta)$.

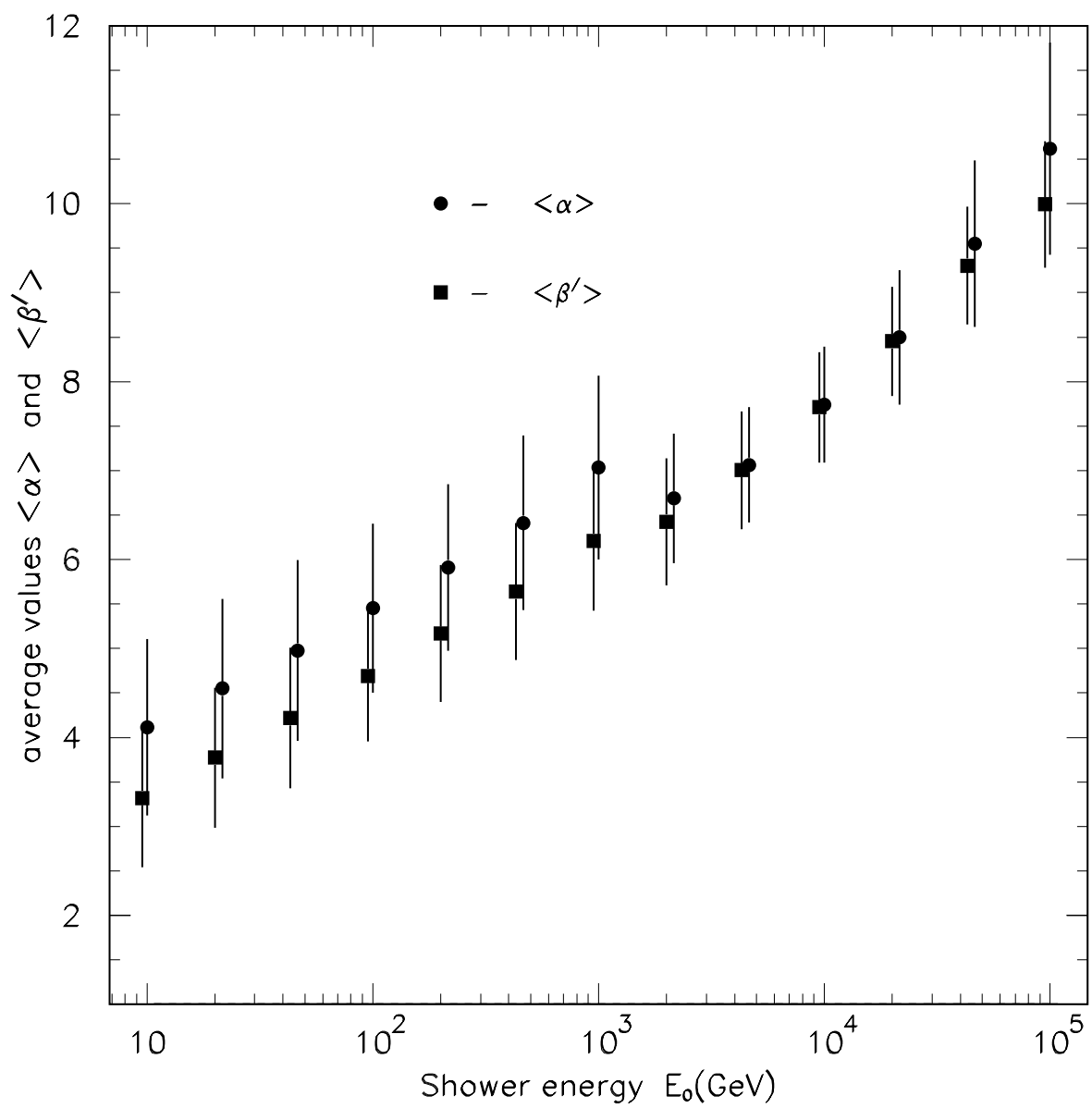


Fig.1

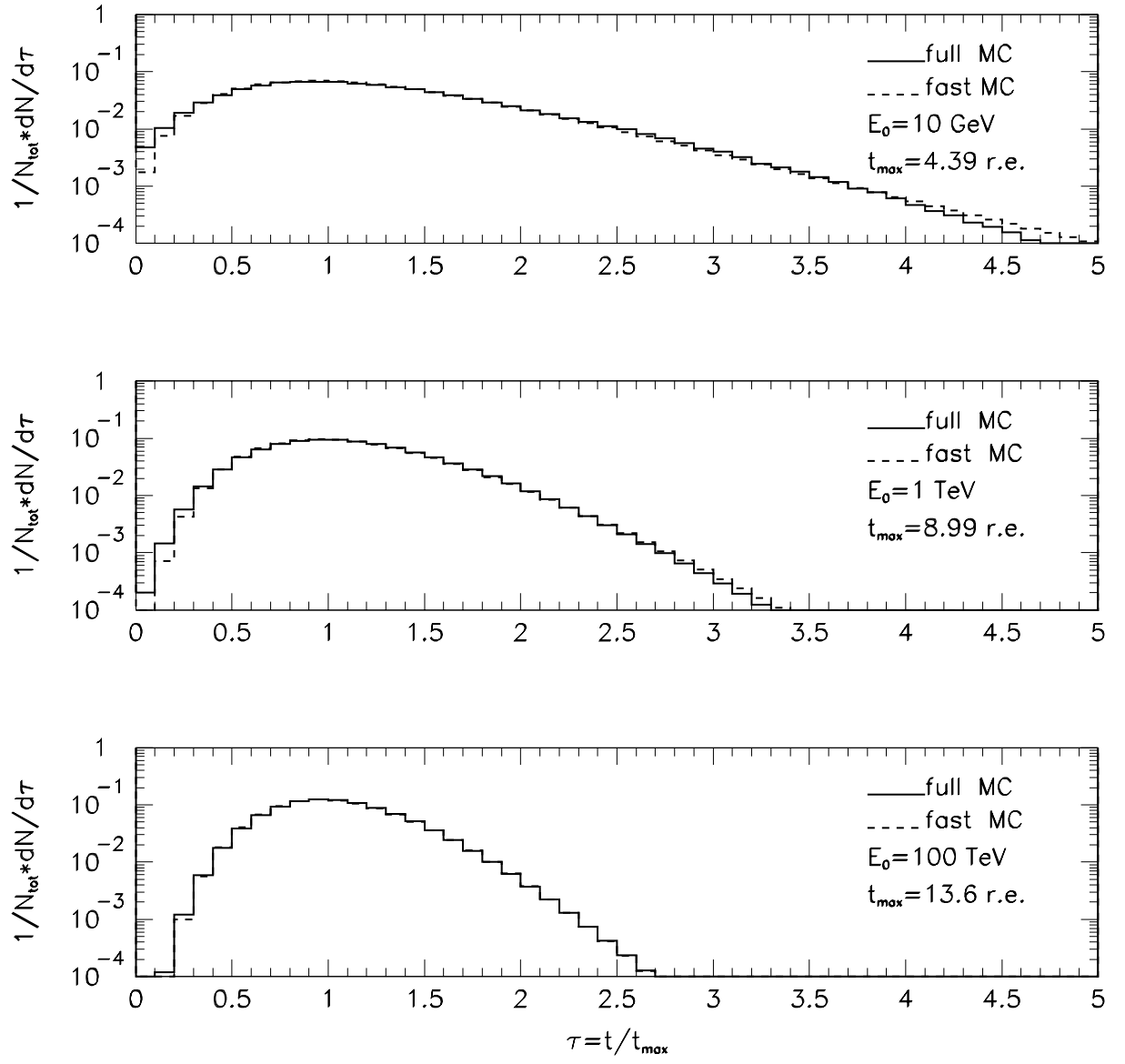


Fig.2

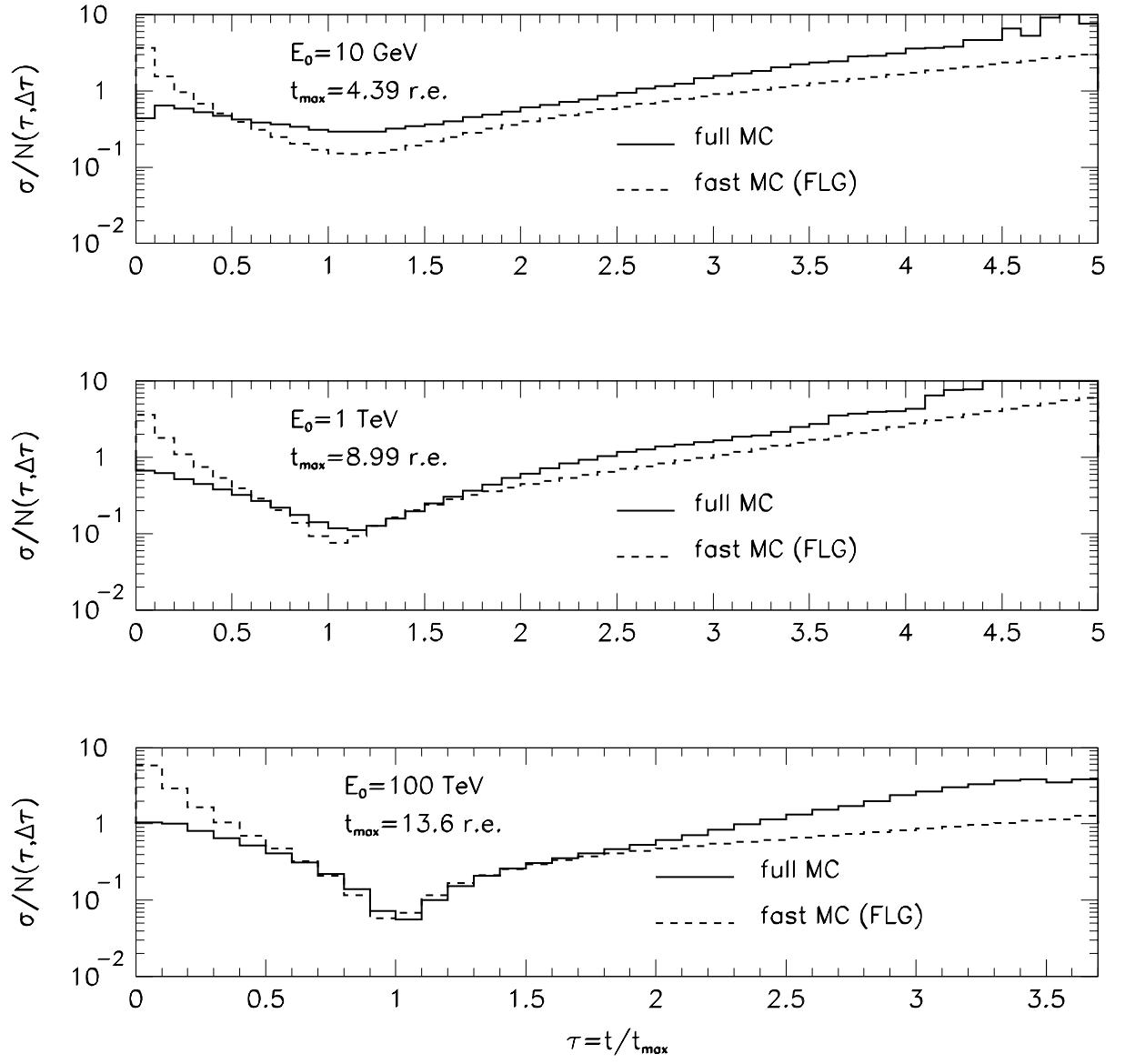


Fig.3

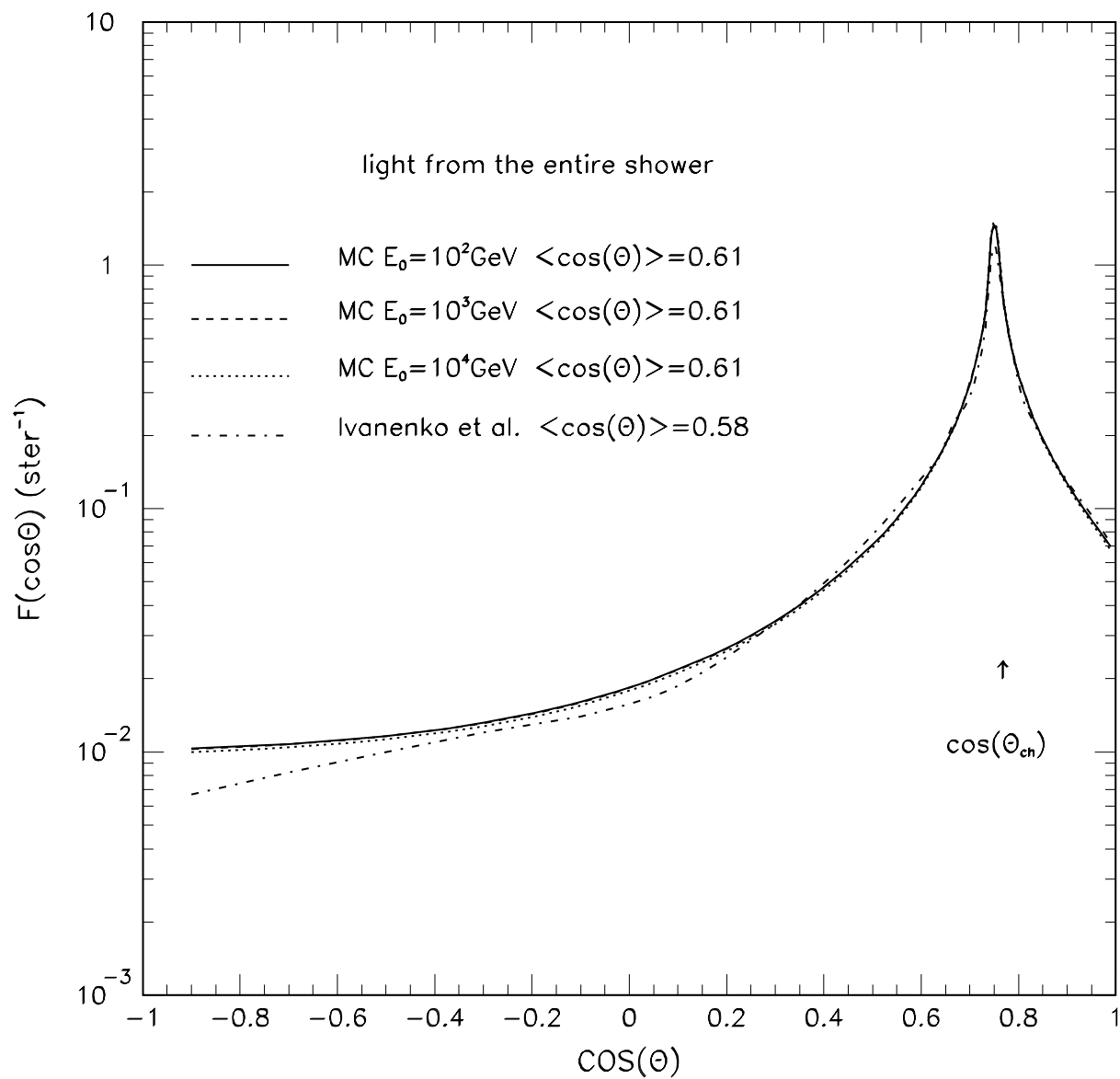


Fig.4

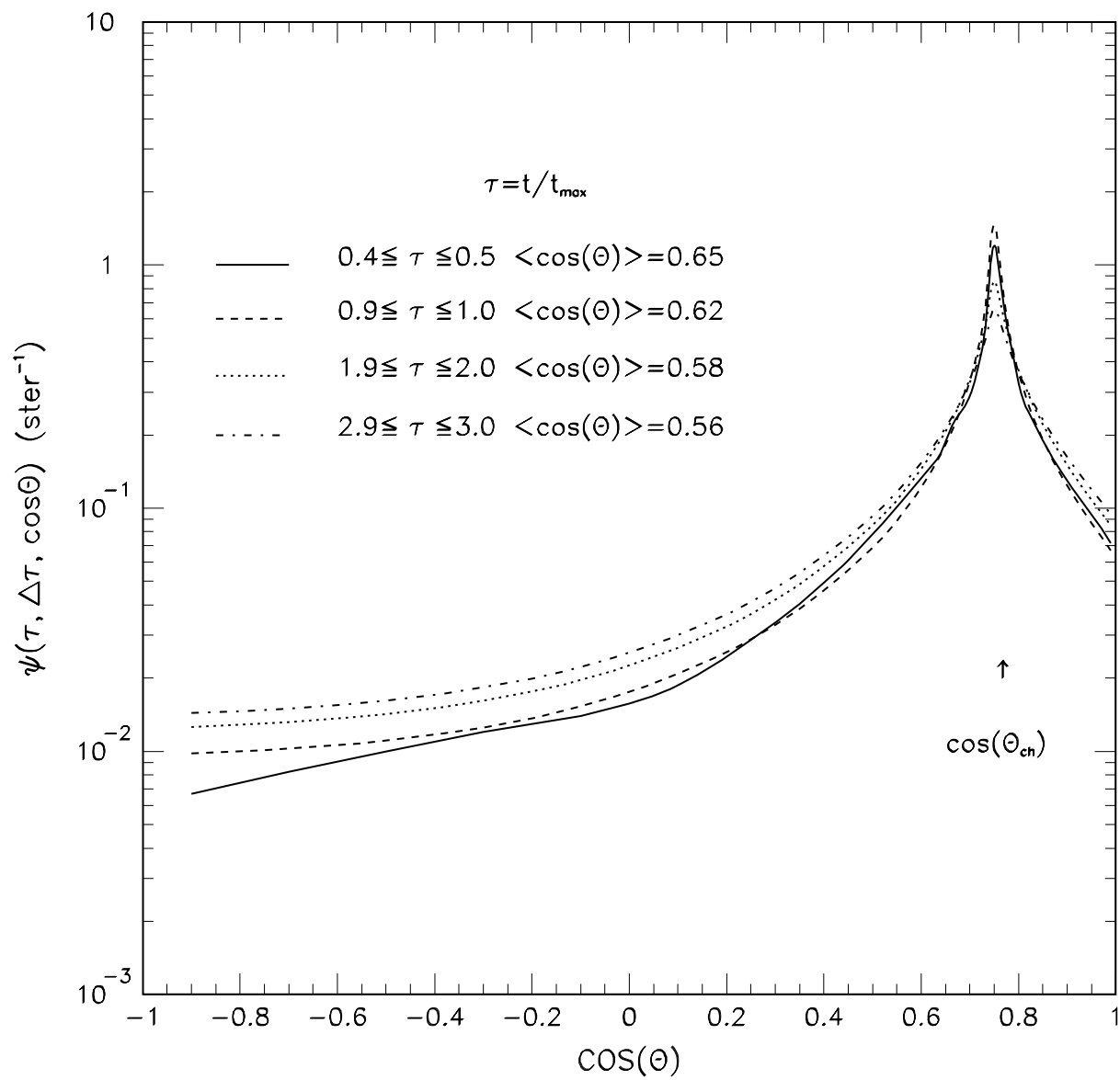


Fig.5

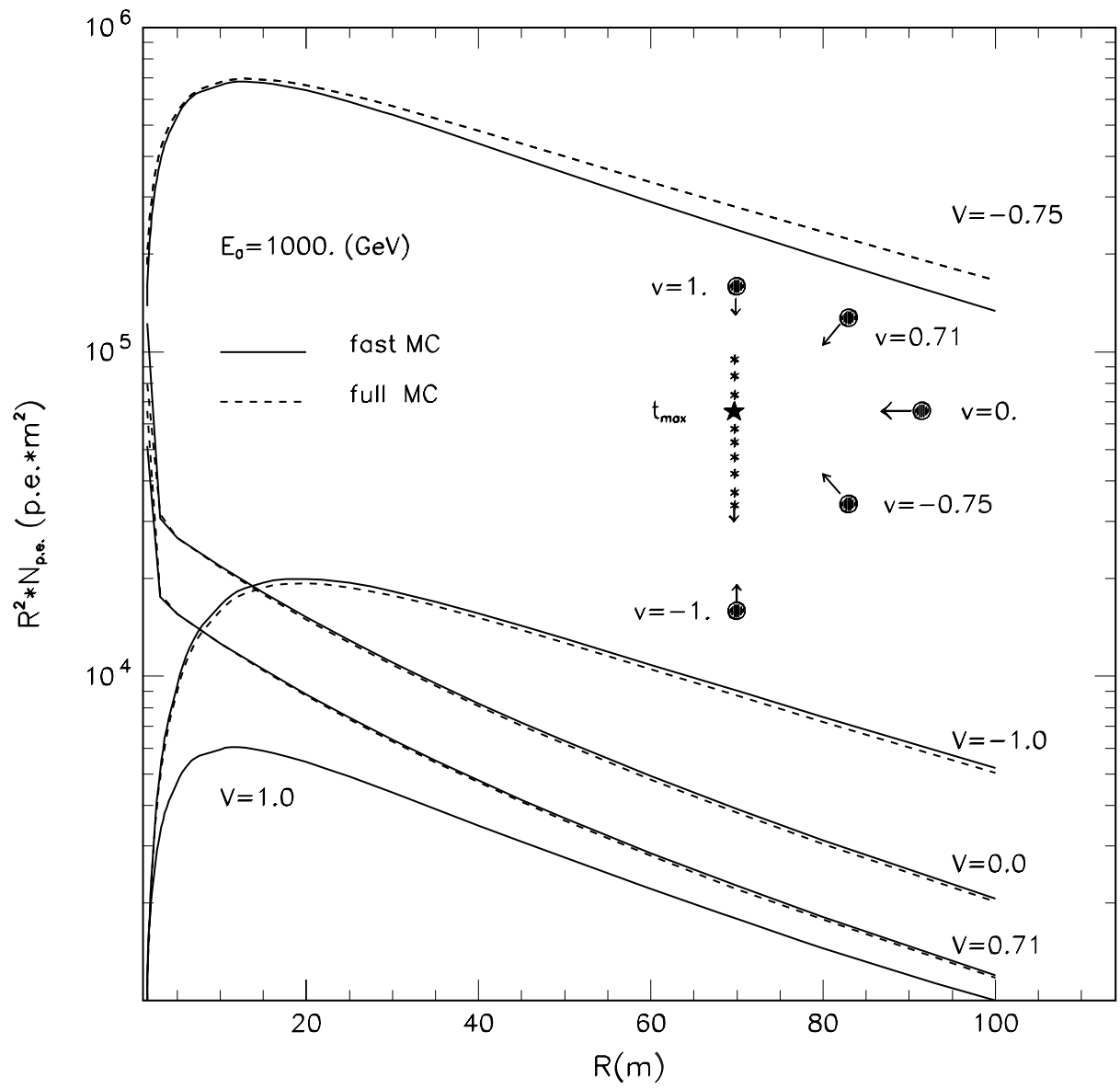


Fig.6